

**FEMA 440**  
**IMPROVEMENT OF NONLINEAR STATIC**  
**SEISMIC ANALYSIS PROCEDURES**

Prepared by:



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# Preface

Knowledgeable engineers have long recognized that the response of buildings to strong ground shaking caused by earthquakes results in inelastic behavior. Until recently, most structural analysis techniques devised for practical application relied on linear procedures to predict the seismic behavior of buildings. With the publication of the ATC-40 Report, *Seismic Evaluation and Retrofit of Concrete Buildings*, in 1996, and the FEMA 273 Report, *Guidelines for the Seismic Rehabilitation of Buildings*, in 1997, nonlinear static analysis procedures became available to engineers providing efficient and transparent tools for predicting seismic behavior of structures.

Both the ATC-40 and FEMA 273 documents present similar performance-based engineering methods that rely on nonlinear static analysis procedures for prediction of structural demands. While procedures in both documents involve generation of a “pushover” curve to predict the inelastic force-deformation behavior of the structure, they differ in the technique used to calculate the inelastic displacement demand for a given ground motion. The FEMA 273 document uses the Coefficient Method, whereby displacement demand is calculated by modifying elastic predictions of displacement demand. The ATC-40 Report details the Capacity-Spectrum Method, whereby modal displacement demand is determined from the intersection of a capacity curve, derived from the pushover curve, with a demand curve that consists of the smoothed response spectrum representing the design ground motion, modified to account for hysteretic damping effects.

The publication of the FEMA 273 and ATC-40 documents resulted in the widespread use of these two methods, and engineers have since reported that the two procedures often give different estimates for displacement demand for the same building. Hence the Applied Technology Council (ATC) proposed to the Federal Emergency Management Agency in 2000 that a study be conducted to determine the reasons for differing results and to develop guidance for practicing engineers on improved application of these two methods. FEMA agreed to fund the investigation, and in October 2000, ATC commenced a project to provide guidance for improved applications of these two widely used inelastic seismic analysis procedures (ATC-55 Project).

The ATC-55 Project had two objectives: (1) the development of practical recommendations for improved prediction of inelastic structural response of buildings to earthquakes (i.e., guidance for improved application of inelastic analysis procedures) and (2) the identification of important issues for future research. Intended outcomes of the project included:

1. Improved understanding of the inherent assumptions and theoretical underpinnings of existing and proposed updated inelastic analysis procedures.
2. Recognition of the applicability, limitations, and reliability of various procedures.
3. Guidelines for practicing engineers to apply the procedures to new and existing buildings.
4. Direction for researchers on issues for future improvements of inelastic analysis procedures.

The project was conducted in three phases over a 3-year time span. Phase 1 consisted of the assembly and refinement of important issues relating to the improvement of inelastic seismic analysis procedures. Activities included (1) the solicitation of input from researchers and practicing engineers, and (2) the development of study models of typical buildings to stimulate discussion, facilitate analytical studies, and provide example applications. Phase 2 consisted of analytical studies to explore selected key issues, the generation of written discussions on important topics, and the development of examples of the application of inelastic analysis procedures. This phase also included assembly of guidelines for the improved practical implementation of the procedures. Phase 3 consisted of the report development process, under which this document was drafted, reviewed, and finalized.

This report (FEMA 440) is the final and principal product of the ATC-55 Project. The document has three specific purposes: (1) to provide guidance directly applicable to the evaluation and design of actual structures by engineering practitioners; (2) to facilitate a basic conceptual understanding of underlying principles as well as the associated capabilities and limitations of the procedures; and (3) to provide additional detailed information used in the development of the document for future reference and use by researchers and others.

A wide variety of personnel, with varying capabilities and expertise, participated in the project. The project

was conducted under the direction of ATC Senior Consultant Craig Comartin, who served as Project Director. Technical and management direction were provided by a Project Management Committee consisting of Craig Comartin (Chair), Christopher Rojahn (Ex-Officio member), Ronald O. Hamburger, William T. Holmes, Wilfred D. Iwan, Jack P. Moehle and Jonathan Stewart. A Project Review Panel, identified by ATC with input from FEMA, provided overview and guidance; this Panel consisted of Anthony B. Court (ATC Board Representative), Leonard Joseph, Daniel Shapiro, Steve Sweeney, Chi-Ming Uang, and Michael Valley.

The Project Management Committee created four Focus Groups to assist in developing findings on the following specific subtopics: (1) displacement modification; (2) equivalent linearization; (3) multi-degree-of-freedom effects; and (4) response of short-period buildings, with a specific focus on soil-structure interaction. The purpose of the Focus Groups was to gather fresh perspective from qualified sources that were not directly responsible for the project planning or the resulting recommendations. Focus Group participants reviewed draft materials developed by the project team. They then attended a one-day meeting with representative members of the Project Management Committee and the project team members responsible for the subject materials. The meetings allowed for a constructive discussion of the subject in general and critical feedback – positive and negative – on the draft materials. Focus Group members were also afforded an opportunity to comment on the final draft of materials related to their area of expertise. It is important to note that Focus Group members were not asked to endorse the project process or the recommendations in documents developed as part of the ATC-55 Project. These remain the responsibility of ATC and the Project Management Committee

Each Focus Group consisted of three members. John Hooper, Gregory A. MacRae, and Stephen A. Mahin

were members of the Focus Group on Displacement Modification. The Focus Group on Equivalent Linearization consisted of Terrance Paret, Graham Powell, and Andrew S. Whittaker. Anil K. Chopra, Jon A. Heintz, and Helmut Krawinkler served on the Focus Group on Multi-Degree-of-Freedom Effects, and Jacobo Bielak, Gregory L. Fenves, and James Malley served on the Focus Group on Soil-structure Interaction.

Detailed work on the project was carried out by several Working Groups appointed by the Project Management Committee. The Phase 1 Project Working Group consisted of Joseph R. Maffei (Group Leader), Mark Aschheim, Maureen Coffey, and Mason T. Walters. The Phase 2 Project Working Group consisted of Sinan Akkar, Mark Aschheim, Andrew Guyader, Eduardo Miranda, Junichi Sakai, Jorge Ruiz-Garcia and Tony Yang. Peter N. Mork produced and formatted the electronic files from which this report was printed.

The affiliations of the project personnel identified above are provided in the list of Project participants.

The Applied Technology Council gratefully acknowledges the cooperation, insight and patience provided by the FEMA Project Officer, Michael Mahoney, and the FEMA Technical Monitor, Robert D. Hanson. ATC also gratefully acknowledges the National Science Foundation for supplemental funding provided through the Pacific Earthquake Engineering Research Center to conduct the investigation of the response of short-period buildings, soil-structure-foundation interaction, and application of the proposed methods. NSF also provided funding for the research of Andrew Guyader on equivalent linearization. A NATO science fellowship from the Scientific Research and Technical Council of Turkey provided partial support for research by Sinan Akkar.

Christopher Rojahn  
ATC Executive Director

# Executive Summary

This document records in detail an effort to assess current nonlinear static procedures (NSP) for the seismic analysis and evaluation of structures. In addition, the document presents suggestions that were developed to improve these procedures for future application by practicing engineers. The elements of work included several analytical studies to evaluate current procedures and to test potential improvements. An extensive review of existing pertinent technical literature was compiled. A survey of practicing engineers with experience in applying nonlinear static procedures was also conducted. Expert practitioners and researchers in appropriate fields worked together to develop the proposed improvements presented in this document. The context for the work was provided by three existing documents, the FEMA 273 *Guidelines for the Seismic Rehabilitation of Buildings*, the successor FEMA 356 *Prestandard and Commentary for the Seismic Rehabilitation of Buildings*, and the ATC-40 report, *Seismic Evaluation and Retrofit of Concrete Buildings*, each of which contain procedures for nonlinear static analysis. These procedures were both evaluated and suggestions for improvement are made for each. Not all of the portions of the two current documents (FEMA 356 and ATC-40) were evaluated. Conclusions regarding the relative accuracy or technical soundness of these documents should not be inferred beyond the specific material and discussions contained in this document.

## 1. Overview of Inelastic Seismic Analysis Procedures

Nonlinear static procedures are one type of inelastic analysis that can be used to estimate the response of structures to seismic ground shaking. The differences between the various approaches relate to the level of detail of the structural model and the characterization of the seismic ground shaking. Detailed structural models can often be simplified into equivalent multi-degree-of-freedom (MDOF) models; or, in some cases, single-degree-of-freedom (SDOF) oscillator models, as with nonlinear static procedures. The most detailed characterizations of seismic ground motion are actual ground motion records that comprise accelerations, velocities, and displacements expected at the ground surface at a specific site. A simplification can be made by representing the effects ground motion has in the frequency domain with response spectra that plot maximum response of an elastic SDOF oscillator as a

function of period. This is the type of characterization normally used for nonlinear static procedures.

The discussion provided in Chapter 2 includes basic descriptions of the two nonlinear static procedures that currently are used in practice. FEMA 356 utilizes a displacement modification procedure (Coefficient Method) in which several empirically derived factors are used to modify the response of a single-degree-of-freedom model of the structure assuming that it remains elastic. The alternative Capacity-Spectrum Method of ATC-40 is actually a form of equivalent linearization. This technique uses empirically derived relationships for the effective period and damping as a function of ductility to estimate the response of an equivalent linear SDOF oscillator.

## 2. Evaluation of Current Nonlinear Static Procedures

In practice, the current procedures can result in estimates of maximum displacement that are significantly different from one another. This has caused concern on the part of practicing engineers. One of the major objectives of the project was to ascertain the reason for these differences and to try to correct both procedures to produce similar results. Chapter 3 documents a comprehensive evaluation of both current procedures. The basic technique was to develop a series of nonlinear single-degree-of-freedom oscillators of varying period, strength, and hysteretic behavior. These were subjected to ground motion representing different site soil conditions. The resulting database of approximately 180,000 predictions of maximum displacement was used as a benchmark to judge the accuracy of the approximate nonlinear static procedures. This was accomplished by comparing the estimates for each oscillator from both nonlinear static procedures to the results of the nonlinear response history analyses. Differences in the two estimates were compiled and compared in a statistical study.

## 3. Strength Degradation

The results of the evaluation of the nonlinear static procedures suggest that both procedures would benefit from greater clarity with respect to the different types of possible degradation in structures subject to seismic shaking. This is particularly critical for degradation in strength. Chapter 4 presents a discussion of the differences between the consequences of strength loss

within a single cycle of deformation (in-cycle) and that which occurs in subsequent cycles (cyclic). In-cycle strength degradation, including that associated with  $P-\Delta$  effects, can lead to dynamic instability. To account for this, a limitation on the strength of a structure is suggested for use with nonlinear static procedures. The limit is a function of the period of the structure and the post-elastic stiffness characteristics as modified for in-cycle strength degradation. If the structure has less strength than the limit, nonlinear dynamic analysis is recommended.

#### 4. Improved Procedures for Displacement Modification

Based on the evaluation of nonlinear static procedures, Chapter 5 proposes modifications to the Coefficient Method of FEMA 356. The suggestions relate primarily to the coefficients themselves. Improved relationships for coefficients  $C_1$  and  $C_2$  are proposed. It is also suggested that the coefficient  $C_3$  be replaced with a limitation on minimum strength as suggested in the previous section.

#### 5. Improved Procedures for Equivalent Linearization

Chapter 6 presents the results of an effort to improve the practical application of equivalent linearization procedures. The resulting suggestions focus upon improved estimates of equivalent period and damping. This chapter also includes an optional adjustment to generate a modified acceleration-displacement response spectrum (MADRS) that does intersect the capacity spectrum at the Performance Point. Similar to the current ATC-40 procedure, the effective period and damping are both dependent on ductility and consequently an iterative or graphical technique is required to calculate the Performance Point. Several options are outlined in Chapter 6. In application, the improved procedures are similar to the current ATC-40 Capacity-Spectrum Method.

#### 6. Evaluation and Comparison of Improved Nonlinear Static Procedures

The improved procedures were evaluated in an independent study. This study, summarized in Chapter 7, utilized nine elastic-perfectly-plastic oscillators with three different periods and three different strengths. These were subjected to thirteen ground motions for class C sites. Estimates of

maximum displacements were calculated utilizing both current procedures and the proposed improved procedures.

This study was not comprehensive enough to make broad general conclusions. However, a number of key observations can be made:

- The improved procedures do not exhibit large differences between displacement modification and equivalent linearization approaches.
- The improved procedures also produced more accurate estimates of displacements when compared to response history analysis (also known as time-history analysis) results than those produced by the current nonlinear procedures.
- Improved procedures also seem to work well, at least for the case that was studied, in estimating maximum displacement response in conjunction with a design spectrum.
- The results of the evaluation of the improved nonlinear procedures illustrate the dispersion of results from nonlinear response history analysis using design level ground motions.

#### 7. Soil-Structure Interaction Effects

Chapter 8 presents procedures to incorporate soil-structure interaction (SSI) into nonlinear static analyses. The objective is to replace the judgmental limits with rational technical justifications for reducing seismic demand. These SSI techniques address the following issues.

- radiation and material damping in supporting soils;
- full and partial basements; and
- incoherent input to buildings with relatively large plan dimensions.

The basic principals used for the development of the SSI procedures in Chapter 8 have been included in the FEMA 368 *NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures* for the linear analysis and design of new buildings for a number of the years. They have been adapted for use with inelastic procedures. They are applicable to both the displacement modification and equivalent linearization forms of nonlinear static analysis.

## 8. Multiple-Degree-of Freedom Effects

Chapter 9 reviews the accuracy and practical implications of the requirements of ATC-40 and FEMA 356 related to MDOF effects including:

1. current options for load vectors, and
2. the conversion of a MDOF pushover curve to an equivalent SDOF system.

The results of a comprehensive study of five example buildings that examines the differences in response predicted using various options compared to a common nonlinear dynamic analyses benchmark are also summarized. The results are consistent with previous research. Practical implications are:

- Nonlinear static procedures generally provide reliable estimates of maximum floor and roof displacements.
- Nonlinear static procedures are not particularly capable, however, of accurate prediction of maximum drifts at each level, particularly within flexible structures.
- Nonlinear static procedures are very poor predictors of story forces, including shears and overturning moments.
- The use of the first mode load vector is suggested due to the relatively good displacement estimates made with this assumption.
- Multi-mode pushover analysis consisting of the use of multiple load vectors proportional to the mode shapes of the structure and combining them statistically shows promise in producing better estimates in inter-story drifts over the heights of the buildings.
- The provisions of FEMA 356 as to when higher modes are to be considered significant are not particularly reliable.

- Specific limitations as to when nonlinear static procedures produce reliable results are very elusive.
- As a result of the study it was observed that, in many cases, a single time history response of a multi-degree-of-freedom model gave better indications of drifts and story forces than any of the approximate single-degree-of-freedom estimates.

## 9. Important Future Developments

The proposed improvements to nonlinear static analysis procedures in this document will lead to better results in practice. Nonetheless, not all of the shortcomings of NSP's have been addressed. In developing the improvements a number of important observations about the need for future develop and improvement of inelastic seismic analysis procedures have emerged. These include the need for additional developmental work on:

1. Nonlinear Modeling for Cyclic and In-Cycle Degradation of Strength and Stiffness
2. Soil and Foundation Structure Interaction
3. Nonlinear Multi-Degree of Freedom Simplified Modeling

## 10. Application Example

Chapter 10 includes an example application of the recommended nonlinear static analysis procedures on an example building. The application example includes a flowchart describing the implementation process, along with building plans, calculations, and commentary. The example illustrates both the displacement modification and the equivalent linearization procedures to estimate the maximum displacement of a building model.





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