



Effects of Strength and Stiffness Degradation on Seismic Response

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Effects of Strength and Stiffness Degradation on Seismic Response

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Foreword

One of the primary goals of the Federal Emergency Management Agency (FEMA) and the National Earthquake Hazards Reduction Program (NEHRP) is to encourage design and construction practices that address the earthquake hazard and minimize the potential damage resulting from that hazard. This document, *Effects of Strength and Stiffness on Degradation on Seismic Response* (FEMA P440A), is a follow-on publication to *Improvement of Nonlinear Static Seismic Analysis Procedures* (FEMA 440). It builds on another FEMA publication addressing the seismic retrofit of existing buildings, the *Prestandard and Commentary for Seismic Rehabilitation of Buildings* (FEMA 356) and the subsequent publication, ASCE/SEI Standard 41-06 *Seismic Rehabilitation of Existing Buildings* (ASCE 41).

The goal of FEMA 440 was improvement of nonlinear static analysis procedures, as depicted in FEMA 356 and ASCE 41, and development of guidance on when and how such procedures should be used. It was a resource guide for capturing the current state of the art in improved understanding of nonlinear static procedures, and for generating future improvements to those products. One of the recommendations to come out of that work was to fund additional studies of cyclic and in-cycle strength and stiffness degradation, and their impact on response and response stability.

This publication provides information that will improve nonlinear analysis for cyclic response, considering cyclic and in-cycle degradation of strength and stiffness. Recent work has demonstrated that it is important to be able to differentiate between cyclic and in-cycle degradation in order to more accurately model degrading behavior, while current practice only recognizes cyclic degradation, or does not distinguish between the two. The material contained within this publication is expected to improve nonlinear modeling of structural systems, and ultimately make the seismic retrofit of existing hazardous buildings more cost-effective.

This publication reaffirms FEMA's ongoing efforts to improve the seismic safety of new and existing buildings nationwide. This project is an excellent example of the interagency cooperation that is made possible through the NEHRP. FEMA is proud to have sponsored the development of this resource document through the Applied Technology Council (ATC), and is grateful

for work done by the Project Technical Director, Craig Comartin, the Project Management Committee, the Project Review Panel, the Project Working Group, and all other contributors who made this publication possible. All those who participated are listed at the end of this document, and FEMA appreciates their involvement.

Federal Emergency Management Agency

Preface

In September 2004 the Applied Technology Council (ATC) was awarded a “Seismic and Multi-Hazard Technical Guidance Development and Support” contract (HSFEHQ-04-D-0641) by the Federal Emergency Management Agency (FEMA) to conduct a variety of tasks, including one entitled “Advanced Seismic Analysis Methods – Resolution of Issues” (ATC-62 Project). The purpose of this project was to resolve a series of difficult technical issues that were identified during the preparation of the FEMA 440 report, *Improvement of Nonlinear Static Seismic Analysis Procedures* (FEMA, 2005).

FEMA 440 was funded by FEMA to develop improvements to nonlinear static analysis procedures contained in the FEMA 356 *Prestandard and Commentary for the Seismic Rehabilitation of Buildings* (FEMA, 2000), and the ATC-40 Report, *Seismic Evaluation and Retrofit of Concrete Buildings* (ATC, 1996). Unresolved technical issues identified in FEMA 440 included the need for additional guidance and direction on: (1) component and global modeling to consider nonlinear degrading response; (2) soil and foundation-structure interaction modeling; and (3) simplified nonlinear multiple-degree-of-freedom modeling.

Of these issues, this project has investigated nonlinear degrading response and conducted limited initial studies on multiple-degree-of-freedom effects. Work has included an extensive literature search and review of past studies on nonlinear strength and stiffness degradation, and review of available hysteretic models for capturing degrading strength and stiffness behavior. To supplement the existing body of knowledge, focused analytical studies were performed to explore the effects of nonlinear degradation on structural response. This report presents the findings and recommendations resulting from these efforts.

ATC is indebted to the members of the ATC-62 Project Team who participated in the preparation of this report. Direction of technical activities, review, and development of detailed recommendations were performed by the Project Management Committee, consisting of Craig Comartin (Project Technical Director), Eduardo Miranda, and Michael Valley. Literature reviews and focused analytical studies were conducted by Dimitrios Vamvatsikos. Technical review and comment at critical developmental

stages were provided by the Project Review Panel, consisting of Kenneth Elwood, Subhash Goel, and Farzad Naeim. A workshop of invited experts was convened to obtain feedback on preliminary findings and recommendations, and input from this group was instrumental in shaping the final product. The names and affiliations individuals who contributed to this work are included in the list of Project Participants provided at the end of this report.

ATC also gratefully acknowledges Michael Mahoney (FEMA Project Officer), Robert Hanson (FEMA Technical Monitor), and William Holmes (ATC Project Technical Monitor) for their input and guidance in the preparation of this report, Peter N. Mork for ATC report production services, and David Hutchinson as ATC Board Contact.

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Executive Summary

Much of the nation's work regarding performance-based seismic design has been funded by the Federal Emergency Management Agency (FEMA), under its role in the National Earthquake Hazards Reduction Program (NEHRP). Prevailing practice for performance-based seismic design is based on FEMA 273, *NEHRP Guidelines for the Seismic Rehabilitation of Buildings* (FEMA, 1997) and its successor documents, FEMA 356, *Prestandard and Commentary for the Seismic Rehabilitation of Buildings* (FEMA, 2000), and ASCE/SEI Standard 41-06, *Seismic Rehabilitation of Existing Buildings* (ASCE, 2006b). This series of documents has been under development for over twenty years, and has been increasingly absorbed into engineering practice over that period.

The FEMA 440 report, *Improvement of Nonlinear Static Seismic Analysis Procedures* (FEMA, 2005), was commissioned to evaluate and develop improvements to nonlinear static analysis procedures used in prevailing practice. Recommendations contained within FEMA 440 resulted in immediate improvement in nonlinear static analysis procedures, and were incorporated in the development of ASCE/SEI 41-06. However, several difficult technical issues remained unresolved.

1. Project Objectives

The Applied Technology Council (ATC) was commissioned by FEMA under the ATC-62 Project to further investigate the issue of component and global response to degradation of strength and stiffness. Using FEMA 440 as a starting point, the objectives of the project were to advance the understanding of degradation and dynamic instability by:

- Investigating and documenting currently available empirical and theoretical knowledge on nonlinear cyclic and in-cycle strength and stiffness degradation, and their affects on the stability of structural systems
- Supplementing and refining the existing knowledge base with focused analytical studies

- Developing practical suggestions, where possible, to account for nonlinear degrading response in the context of current seismic analysis procedures.

This report presents the findings and conclusions resulting from the literature search and focused analytical studies, and provides recommendations that can be used to improve both nonlinear static and nonlinear response history analysis modeling of strength and stiffness degradation for use in performance-based seismic design.

2. Literature Review

Past research has shown that in-cycle strength and stiffness degradation are real phenomena, and recent investigations confirm that the effects of in-cycle strength and stiffness degradation are critical in determining the possibility of lateral dynamic instability.

The body of knowledge is dominated by studies conducted within the last 20 years; however, relevant data on this topic extends as far back as the 1940s. A summary of background information taken from the literature is provided in Chapter 2. A comprehensive collection technical references on this subject is provided in Appendix A.

3. Focused Analytical Studies

To supplement the existing body of knowledge, focused analytical studies were performed using a set of eight nonlinear springs representing different types of inelastic hysteretic behavior. These basic spring types were used to develop 160 single-spring systems and 600 multi-spring systems with differing characteristics. Each system was subjected to incremental dynamic analysis with 56 ground motion records scaled to multiple levels of increasing intensity. The result is an extensive collection of data on nonlinear degrading response from over 2.6 million nonlinear response history analyses on single- and multi-spring systems.

Development of single- and multi-spring models is described in Chapter 3, analytical results are summarized in Chapter 4, and sets of analytical data are provided in the appendices. A Microsoft Excel visualization tool that was developed to view all available data from multi-spring studies is included on the CD accompanying this report.

4. Comparison with FEMA 440 Limitations on Strength for Lateral Dynamic Instability

In FEMA 440, a minimum strength requirement (R_{max}) was developed as an approximate measure of the need to further investigate the potential for lateral dynamic instability caused by in-cycle strength degradation and P-delta effects. To further investigate correlation between R_{max} and lateral dynamic instability, the results of this equation were compared to quantile incremental dynamic analysis (IDA) curves for selected multi-spring systems included in this investigation. Results indicate that values predicted by the FEMA 440 equation for R_{max} are variable, but generally plot between the median and 84th percentile results for lateral dynamic instability of the systems investigated. Observed trends indicate that an improved equation, in a form similar to R_{max} , could be developed as a more accurate (less variable) predictor of lateral dynamic instability for use in current nonlinear static analysis procedures.

5. Findings, Conclusions, and Recommendations

Findings, conclusions, and recommendations resulting from the literature review and focused analytical studies of this investigation are collected and summarized in Chapter 5, grouped into the following categories:

- Findings related to improved understanding of nonlinear degrading response and judgment in implementation of nonlinear analysis results in engineering practice.
- Recommended improvements to current nonlinear analysis procedures
- Suggestions for further study

6. Findings Related to Improved Understanding and Judgment

Results from focused analytical studies were used to identify predominant characteristics of median incremental dynamic analysis (IDA) curves and determine the effects of different degrading behaviors on the dynamic stability of structural systems. Observed practical ramifications from these studies are summarized below:

- Behavior of real structures can include loss of vertical-load-carrying capacity at lateral displacements that are significantly smaller than those associated with sidesway collapse. Use of the findings of this investigation with regard to lateral dynamic instability (sidesway

collapse) in engineering practice should include consideration of possible vertical collapse modes that could be present in the structure under consideration.

- Historically, the term “backbone curve” has referred to many different things. For this reason, two new terms have been introduced to distinguish between different aspects of hysteretic behavior. These are the *force-displacement capacity boundary*, and *cyclic envelope*.
- Nonlinear component parameters should be based on a force-displacement capacity boundary, rather than a cyclic envelope. Determining the force-displacement capacity boundary from test results using a single cyclic loading protocol can result in overly conservative predictions of maximum displacement.
- Observed relationships between selected features of the force-displacement capacity boundary and the resulting characteristics of median IDA curves support the conclusion that the nonlinear dynamic response of a system can be correlated to the parameters of the force-displacement capacity boundary of that system. Of particular interest is the relationship between global deformation demand and the intensity of the ground motion at lateral dynamic instability (collapse). Results indicate that it is possible to use nonlinear static procedures to estimate the potential for lateral dynamic instability of systems exhibiting in-cycle degradation.
- It is important to consider the dependence on period of vibration in conjunction with the effects of other parameters identified in this investigation. The generalized effect of any one single parameter can be misleading.
- It is important to recognize the level of uncertainty that is inherent in nonlinear analysis, particularly regarding variability in response due to ground motion uncertainty.
- In most cases the effects of in-cycle strength degradation dominate the nonlinear dynamic behavior of a system. This suggests that in many cases the effects of cyclic degradation can be neglected.
- Two situations in which the effects of cyclic degradation were observed to be important include: (1) short period systems; and (2) systems with very strong in-cycle strength degradation effects (very steep negative slopes and very large drops in lateral strength).

7. Improved Equation for Evaluating Lateral Dynamic Instability

An improved estimate for the strength ratio at which lateral dynamic instability might occur (R_{di}) was developed based on nonlinear regression of the extensive volume of data generated during this investigation. In performing this regression, results were calibrated to the median response of the SDOF spring systems studied in this investigation. Since the proposed equation for R_{di} has been calibrated to median response, use of this equation could eliminate some of the conservatism inherent in the current R_{max} limitation on use of nonlinear static procedures. Calibrated using the extensive volume of data generated during this investigation, use of this equation could improve the reliability of current nonlinear static procedures with regard to cyclic and in-cycle degradation.

Median response, however, implies a fifty percent chance of being above or below the specified value. Use of R_{di} in engineering practice should consider whether or not a median predictor represents an appropriate level of safety against the potential for lateral dynamic instability. If needed, a reduction factor could be applied to the proposed equation for R_{di} to achieve a higher level of safety on the prediction of lateral dynamic instability.

8. Simplified Nonlinear Dynamic Analysis Procedure

Focused analytical studies comparing force-displacement capacity boundaries to incremental dynamic analysis results led to the concept of a simplified nonlinear dynamic analysis procedure. In this procedure, a nonlinear static analysis is used to generate an idealized force-deformation curve (i.e., static pushover curve), which is then used as a force-displacement capacity boundary to constrain the hysteretic behavior of an equivalent SDOF oscillator. This SDOF oscillator is then subjected to incremental dynamic analysis, or approximate IDA results are obtained using the open source software tool, *Static Pushover 2 Incremental Dynamic Analysis*, SPO2IDA (Vamvatsikos and Cornell 2006). A Microsoft Excel version of the SPO2IDA application is included on the CD accompanying this report.

The procedure is simplified because only a SDOF oscillator is subjected to nonlinear dynamic analysis. Further simplification is achieved through the use of SPO2IDA, which avoids the computational effort associated with incremental dynamic analysis. This simplified procedure is shown to have several advantages over nonlinear static analysis procedures. Use of the procedure is explained in more detail in the example application contained in Appendix F.

9. Application of Results to Multiple-Degree-of-Freedom Systems

Multi-story buildings are more complex dynamic systems whose seismic response is more difficult to estimate than that of SDOF systems. Recent studies have suggested that it may be possible to estimate the collapse capacity of multiple-degree-of-freedom (MDOF) systems through dynamic analysis of equivalent SDOF systems. As part of the focused analytical work, preliminary studies of MDOF systems were performed. Results indicate that many of the findings for SDOF systems in this investigation (e.g., the relationship between force-displacement capacity boundary and IDA curves; the equation for R_{di}) may be applicable to MDOF systems.

Results of MDOF investigations are summarized in Appendix G. More detailed study of the application of these results to MDOF systems is recommended, and additional investigations are planned under a project funded by the National Institute of Standards and Technology (NIST).

10. Concluding Remarks

Using FEMA 440 as a starting point, this investigation has advanced the understanding of degradation and dynamic instability by:

- Investigating and documenting currently available empirical and theoretical knowledge on nonlinear cyclic and in-cycle strength and stiffness degradation, and their affects on the stability of structural systems
- Supplementing and refining the existing knowledge base with focused analytical studies

Results from this investigation have confirmed conclusions regarding degradation and dynamic instability presented in FEMA 440, provided updated information on modeling to differentiate between cyclic and in-cycle strength and stiffness degradation, and linked nonlinear dynamic response to major characteristics of component and system degrading behavior. This information will ultimately improve nonlinear modeling of structural components, improve the characterization of lateral dynamic instability, and reduce conservatism in current analysis procedures making it more cost-effective to strengthen existing buildings for improved seismic resistance in the future.

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